# Single–photon events in $e^+e^-$ collisions

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## Abstract

We provide a detailed investigation of single–photon production processes in $e^+e^-$ collisions with missing momenta carried by neutrinos or neutralinos. The transition amplitudes for both processes can be organized into a generic simplified, factorized form; each neutral V±A vector current of missing energy carriers is factorized out and all the characteristics of the reaction is solely included in the electron vector current. Firstly, we apply the generic form to give a unified description of a single–photon production with a Dirac–type or Majorana–type neutrino–pair and to confirm their identical characteristics as suggested by the so-called Practical Dirac–Majorana Confusion Theorem. Secondly, we show that the generic amplitude form is maintained with the anomalous P– and C–invariant WWγ couplings in the neutrino–associated process and it enables us to easily understand large contributions of the anomalous WWγ couplings at higher energies and, in particular, at the points away from the Z–resonance peak. Finally, the neutralino–associated process, which receives modifications in both the left–handed and right-handed electron currents due to the exchanges of the left–handed and right-handed selectrons, can be differentiated from the neutrino–associated ones through the left–right asymmetries and/or the circular polarization of the outgoing photon.

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# I. INTRODUCTION

All the large luminosity and high energy experiments up to now have confirmed the validity of the Standard Model (SM) to an unexpectedly high level [1]. In spite of its extraordinary success, the SM has a lot of conceptual problems such as the gauge hierarchy problem so that it is believed to be valid only at the electroweak scale and to be extended at higher energies. The first would-be evidence beyond the SM, although it has to be independently confirmed by other experiments, has come from the neutrino sector as the zenith-angle-dependent neutrino flux has been observed in the Super–Kamiokande experiment [2]. On the other hand, high energy collider experiments such as LEP2, LHC and a high energy $e^+e^-$ linear collider (NLC) should accelerate a broad investigation of new physics beyond the SM in the near future.

The process $e^+e^- \rightarrow \gamma + X$ with a distinctive "photon–plus–missing–energy" signal can serve as one of the most efficient processes for the exploration of new physics. In the process the missing energy can be carried by the SM neutrinos or weakly interacting or invisible new (s)particles. In the framework of the SM, the single–photon process with the missing energy carried by neutrinos has been exploited to count the number of light neutrino species at PETRA, SLAC and LEP1 [3,4] since, at low energies, the contribution from the $t$–channel W–exchange diagrams becomes negligible. However, the W–exchange contributions become important at high energies so that the neutrino–associated single–photon process allows for measuring the WW$\gamma$ coupling independently of the WWZ coupling unlike the most discussed $e^+e^- \rightarrow W^+W^-$.

The events with a photon plus missing energy in $e^+e^-$ collisions might originate from other mechanisms[1], signaling new physics beyond the SM. For example, such final states can be produced in the Minimal Supersymmetric SM (MSSM), one of the most promising frameworks for the new theory. The missing energy in these events is caused by the weakly interacting or invisible particles such as lightest neutralinos, gravitinos and/or sneutrinos. In all such cases the SM neutrino-associated single–photon events are irreducible background. Therefore, in order to reach a definite conclusion of new physics, comprehensive calculations and reliable estimations of all possible single–photon processes are requisite.

In the present work, we provide a unified description of the following three cases for single–photon events: (i) $e^+e^- \rightarrow \gamma\nu\nu$ in the SM including the case when the neutrinos are of Majorana type, (ii) $e^+e^- \rightarrow \gamma\nu\nu$ with the P– and C–preserving general WW$\gamma$ coupling, and (iii) $e^+e^- \rightarrow \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ in the MSSM assuming that the lightest neutralino is the lightest supersymmetric particle (LSP). Several diagrams are involved in all the processes under consideration so that the complete calculations look quite demanding. However, as will be shown in the following, the transition amplitude of every single–photon process is organized into a generic simple, unified form; each neutral vector current of missing energy carriers is factorized out and all the dynamical characteristics for the process are solely included in the

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[1]Recent developments [5] in superstring theory have led to a radical rethinking of the possibilities for new particles and dynamics arising from extra compactified spatial dimensions. Among the new particle states, the so–called Kaluza–Klein massive gravitons [6] can be the invisible particles carrying the missing energy in the single–photon events.

electron vector current.

The rest of the present work is organized as follows. In Section II, we exemplify the amplitude reduction procedure for the neutrino–associated single–photon process in the SM and apply it to give a unified description of a single–photon production with a Dirac–type or Majorana–type neutrino–pair, which facilitate confirming the indistinguishability between the observations of both processes as suggested by the so-called Practical Dirac–Majorana Confusion Theorem [7]. Then, we show that the generic amplitude form is maintained even after including P– and C–preserving anomalous $WW\gamma$ couplings in the neutrino–associated process and the simplified form clearly exhibits large contributions of the anomalous $WW\gamma$ couplings at higher energies and, in particular, at the points away from the Z–resonance peak. In Section III we consider the neutralino–associated single–photon process in the MSSM. This process involves modifications in both the left–handed and right-handed electron currents due to the left–handed and right-handed selectron exchanges. Nevertheless, the simple unified form of the amplitude, which appears in the neutrino–associated single–photon process, can be also applied to the process with two identical neutralinos of Majorana type as final missing–energy states. Section IV is devoted to assessing the usefulness of the left–right asymmetry and the circular polarization of the outgoing photon in distinguishing the neutralino–associated process from the neutrino–associated one. Finally, we reserve Section V for the summary and conclusions.

## II. NEUTRINO–ASSOCIATED SINGLE–PHOTON PROCESSES

### A. Amplitude reduction

In this subsection, we describe how to obtain a simple unified amplitude for the processes with a distinctive photon–plus–missing-energy through the following specific example [8]:

$$e^-(p_1) + e^+(p_2) \rightarrow \gamma(k) + \nu(k_1) + \bar{\nu}(k_2) \tag{1}$$

The neutrino–associated single–photon process (1) involves five Feynman diagrams in the SM; three W-mediated and two Z-mediated ones as shown in Fig. 1. The application of the Fierz rearrangement formulas

$$\left[\bar{\psi}_1 \gamma_\mu P_L \psi_2\right] \left[\bar{\psi}_3 \gamma^\mu P_L \psi_4\right] = -\left[\bar{\psi}_1 \gamma_\mu P_L \psi_4\right] \left[\bar{\psi}_3 \gamma^\mu P_L \psi_2\right],$$

$$\left[\bar{\psi}_1 P_R \psi_2\right] \left[\bar{\psi}_3 P_L \psi_4\right] = \frac{1}{2}\left[\bar{\psi}_1 \gamma_\mu P_L \psi_4\right] \left[\bar{\psi}_3 \gamma^\mu P_R \psi_2\right], \tag{2}$$

to the three W–mediated diagrams reduces the production amplitude to a general form

$$\mathcal{M} = \frac{eg^2}{2\cos^2\theta_W} \frac{g_{ZXX}}{(k_1 + k_2)^2 - m_Z^2} \left[\bar{u}(k_1)\gamma_\mu P_L v(k_2)\right]$$

$$\times \bar{v}_e(p_2)\left[\frac{\gamma^\mu(\not{\varepsilon}^* \not{k} + 2p_1 \cdot \varepsilon^*)}{2p_1 \cdot k}\{L_1 P_L + R_1 P_R\}\right.$$

$$+ \frac{(\not{\varepsilon}^* \not{k} - 2p_2 \cdot \varepsilon^*)\gamma^\mu}{2p_2 \cdot k}\{L_2 P_L + R_2 P_R\}$$

$$\left. + A_L^{\mu\nu}\gamma_\nu P_L + A_R^{\mu\nu}\gamma_\nu P_R \right]u_e(p_1). \tag{3}$$

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Here, $P_{L,R} = (1 \mp \gamma_5)/2$, the parameter $g_{ZXX}$ denotes the normalized coupling strength of the $ZXX$ vertex (e.g. $g_{Z\nu\bar{\nu}} = 1$) and $\varepsilon^*$ the polarization vector of the outgoing photon. Since the $We\nu$ vertex is of the left–handed type in the SM, only the left–handed form factors are affected by the W–exchange diagrams but the right–handed ones are exclusively determined by the $Zee$ vertex:

$$L_i = \epsilon_L + [2p_i \cdot (k + k_i) + m_W^2]f_W \,, \quad R_i = \epsilon_R \ \ [i = 1, 2]\,,$$
$$A_L^{\mu\nu} = 2g^{\mu\nu}(k_2 - p_1) \cdot \varepsilon^* f_W \,, \qquad\qquad A_R^{\mu\nu} = 0 \,, \tag{4}$$

where $\epsilon_L$ and $\epsilon_R$ are the SM left- and right-handed couplings for the $Zee$ vertex and $f_W$ is the momentum-dependent form factor:

$$\epsilon_L = -\frac{1}{2} + \sin^2\theta_W \,, \quad \epsilon_R = \sin^2\theta_W$$
$$f_W = -\cos^2\theta_W \frac{2k_1 \cdot k_2 - m_Z^2}{(2p_1 \cdot k_1 + m_W^2)(2p_2 \cdot k_2 + m_W^2)} \,, \tag{5}$$

with the electroweak mixing angle $\theta_W$. Note that the neutrino vector current of the V$-$A form is factored out and the whole dynamical information of the process is included only in the electron vector current. The contributions from the W–mediated processes to the form factors vanish at the Z–resonance pole. The last two terms, $A_L^{\mu\nu}$ and $A_R^{\mu\nu}$, in (3) play a role in conserving $U(1)_{EM}$ gauge invariance and they are proportional to the factor $f_W$.

The expression in eq. (3) is of a very generic form so that it can be applied to the amplitude for any process producing a single photon and a fermion–pair in $e^+e^-$ collisions. This property will be explicitly demonstrated with three examples; (i) the production of a photon and a Majorana neutrino pair, (ii) the case with the anomalous WW$\gamma$ couplings and (iii) the production of a photon and a lightest neutralino pair.

In order to check the validity of the simplified form for the process $e^+e^- \to \gamma\nu\bar{\nu}$, we perform a Monte–Carlo phase–space integration by BASES [9] with the expression in eq. (3) and illustrate in Fig. 2 the dependence of the differential cross section on the photon energy fraction $x_\gamma$ with respect to the electron beam energy $E_b \ [= \sqrt{s}/2]$. Numerically, we find that the differential cross section is completely consistent with that in Ref. [10]. As can be easily checked from the simplified form of the amplitude, the peaks in the differential cross section $d\sigma/dx_\gamma$ are attributed to the $s$-channel Z-mediated diagrams near the photon energy fraction $x_\gamma = 1 - m_Z^2/s$.

### B. Dirac versus Majorana

In the SM, only the neutrinos among fundamental fermions may possess no global discrete quantum numbers such as the lepton numbers, opening the possibility that neutrinos are their own anti-particles, that is to say, Majorana particles. In the light of this aspect, whether light neutrinos are Dirac or Majorana particles has been one of the main issues in neutrino physics. The answer is truly meaningful only when any difference is experimentally observed. In the wide range of neutrino experiments at the colliders, the so–called "Practical Dirac-Majorana Confusion (PDMC) Theorem" in Ref. [7] holds true [11]. Related with the

recent evidence of neutrino oscillation, it will be of particular interest to check the possibility of determining in the neutrino–associated single–photon process whether the produced neutrinos are of Dirac or Majorana type or not.

In principle, there exist some differences at the amplitude level due to different Feynman rules for both types of neutrinos [12]. Compared to Dirac particles, Majorana particles can exhibit two important characteristic features: lepton-number violation and different Feynman rules for interaction vertices involving the Majorana particles. In the reaction $e^+e^- \to \gamma\nu\nu$ for a Majorana neutrino pair, there exists a $u$-channel lepton–number violating diagram corresponding to each $t$-channel lepton–number preserving diagram. Due to the fact that there is no vector current for Majorana fermions, the neutral vector current must be of the type $(\gamma^\mu P_L - \gamma^\mu P_R)$ while the charged vector current remains intact. Nevertheless, we will show that, if the neutrinos are not detected and (almost) massless, the experimental signatures at high–energy colliders are identical for both Dirac and Majorana neutrinos. This is an additional demonstration of the PDMC theorem.

For Majorana neutrinos, the amplitude of each $u$-channel diagram is related to that of corresponding $t$-channel one by

$$\mathcal{M}_u(k_1, k_2) = -\mathcal{M}_t(k_2, k_1),\qquad(6)$$

where the minus sign stems from the interchange of two identical fermions. On the other hand, the neutral vector current of Majorana neutrinos in the Z–mediated diagrams can be expressed in terms of two Dirac–type amplitudes by

$$\bar{u}_M(k_1)\left(\gamma^\mu P_L - \gamma^\mu P_R\right) v_M(k_2) = \bar{u}_M(k_1)\gamma^\mu P_L v_M(k_2) - \bar{u}_M(k_2)\gamma^\mu P_L v_M(k_1),\qquad(7)$$

where we have used Majorana conditions

$$\bar{u}_M(k_1)\gamma^\mu(1 \pm \gamma_5)v_M(k_2) = \bar{u}_M(k_2)\gamma^\mu(1 \mp \gamma_5)v_M(k_1).\qquad(8)$$

As a result, the production amplitude for Majorana neutrinos is expressed by

$$\mathcal{M}_M = \mathcal{M}_D(k_1, k_2) - \mathcal{M}_D(k_2, k_1).\qquad(9)$$

Note that the second term in the right hand side of (9) is the negative of the first term with $k_1^\mu$ and $k_2^\mu$ exchanged. Therefore, the transition amplitude is expressed in terms of two amplitudes which are of the generic form in eq. (3).

We first note that the interference term $\mathcal{M}_D(k_1, k_2)^* \mathcal{M}_D(k_2, k_1)$ in the evaluation of $|\mathcal{M}_M|^2$ becomes, with the help of the expression (3) and the Majorana condition (8),

$$\mathcal{M}_D(k_1, k_2)^* \mathcal{M}_D(k_2, k_1) = \mathcal{E}_{\mu\nu} \sum_{\text{spin}} \bar{v}(k_1)\gamma^\mu P_L u(k_2)\, \bar{u}(k_2)\gamma^\nu P_R v(k_1) = 2m_\nu^2\, g^{\mu\nu}\, \mathcal{E}_{\mu\nu}.\qquad(10)$$

where $\mathcal{E}_{\mu\nu}$ is a covariant tensor composed of the absolute square of the electron vector current. The final term in eq. (10) is obtained by assuming a finite neutrino mass $m_\nu$ and taking the polarization sum. Clearly, when the neutrino mass is negligible compared to the beam energy, the contribution from the interference term vanishes. The practical incapability of explicitly identifying neutrinos at high–energy collider experiments forces us to integrate the differential cross section over the final phase space of neutrinos. As a result, the complete

identity of the integrals of two squared amplitudes over the symmetric phase space of the two neutrino momenta

$$\int d\Phi_2 \, \delta^4(k_1 + k_2 - q) \, |\mathcal{M}_D(k_1, k_2)|^2 = \int d\Phi_2 \, \delta^4(k_1 + k_2 - q) \, |\mathcal{M}_D(k_2, k_1)|^2 \,, \qquad (11)$$

does not leave any difference in the observation of Dirac and Majorana neutrinos.

In summary, any practically observable difference between Dirac and Majorana neutrinos can appear only when neutrinos have a non–negligible mass.

## C. Anomalous WW$\gamma$ coupling

Under the assumption that the discrete symmetries P, C, and T are preserved separately, the general coupling of two charged vector bosons W$^\pm$ with a photon $\gamma$ is derived from the most general and U(1)$_{\mathrm{EM}}$ gauge-invariant Lagrangian [13]

$$\frac{\mathcal{L}_{WW\gamma}}{g} = i\,(W^\dagger_{\mu\nu} W^\mu A^\nu - W^\dagger_\mu A_\nu W^{\mu\nu}) + i\,\kappa_\gamma\, W^\dagger_\mu W_\nu F^{\mu\nu} + \frac{i\lambda_\gamma}{m_W^2} W^\dagger_{\lambda\nu} W^\mu_\nu F^{\mu\lambda} \,, \qquad (12)$$

where $W^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu$ and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. The parameters $\kappa_\gamma$ and $\lambda_\gamma$, [which are 1 and 0 in the SM], are related to the anomalous magnetic dipole moment $\mu_W$ and the electric quadrupole moment $Q_W$ of the W boson by

$$\mu_W = \frac{e(1 + \kappa_\gamma + \lambda_\gamma)}{2m_W} \,,$$

$$Q_W = -\frac{e(\kappa_\gamma - \lambda_\gamma)}{m_W^2} \,. \qquad (13)$$

These self-interactions of gauge bosons have been extensively investigated through various processes at $e^+e^-$ and hadron colliders [14]. Among them the hadron-free reaction $e^+e^- \to \gamma\nu\bar{\nu}$ is favorable in the investigation of the WW$\gamma$ vertex since it does not include the other self interactions of gauge bosons [15]. Even though the gauge group structure of the SM specifies the self interactions of the W, Z and $\gamma$ when regarded as fundamental gauge bosons, their precise confirmation is to be experimentally established [16]. The ALEPH collaboration has reported preliminary results for the coupling $\kappa_\gamma - 1 = 0.05^{+1.2}_{-1.1}$ (stat.) and $\lambda_\gamma = -0.05^{+1.6}_{-1.5}$ (stat.) from the data of the process $e^+e^- \to \gamma\nu\bar{\nu}$ at $\sqrt{s} = 161$, 172, and 183 GeV [17]. Any deviation from the SM prediction will lead to the hint for the theory beyond the SM.

After a little lengthy calculation, we find that even in the existence of the anomalous WW$\gamma$ couplings the transition amplitude for this reaction still keeps the unified form (3) with the following modifications in the form factors:

$$L_i = \epsilon_L + \left[ \left( 1 + \kappa_\gamma - 2\frac{\lambda_\gamma}{m_W^2}(p_i \cdot k_i) \right)(p_i \cdot k) + 2p_i \cdot k_i + m_W^2 \right] f_W \,,$$

$$R_i = \epsilon_R \,,$$

$$A_L^{\mu\nu} = \left[ g^{\mu\nu} \varepsilon^* \cdot (K_2 - K_1) + 2\frac{\lambda_\gamma}{m_W^2}(p_1 - k_1) \cdot k \varepsilon^{*\mu} k^\nu - 2\frac{\lambda_\gamma}{m_W^2}(p_1 - k_1) \cdot \varepsilon^* k^\mu k^\nu \right] f_W \,,$$

$$A_R^{\mu\nu} = 0 \,,$$

$$K_i = k_i + \kappa_\gamma\, p_i - 2\frac{\lambda_\gamma}{m_W^2}(p_i \cdot k_i)\, p_i \,. \qquad (14)$$

Compared to the transition amplitude in the SM, only the V−A part of the electron vector current is modified. This is understandable because the V−A vertex remains the same for the charged electron current with the W boson which is to be coupled with the photon. In Fig. 3, we show the differential cross section with respect to the photon energy fraction $x_\gamma$ at $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV for two cases[2]: three values of $\kappa_\gamma$ with $\lambda_\gamma = 0$ [$\kappa_\gamma = 1, -1.3$ and $3.2$] and three values of $\lambda_\gamma$ with $\kappa_\gamma = 1$ [$\lambda_\gamma = 0, -1$ and $1$]. Note that the effects of non-standard couplings increase at higher energies [18], reflecting the fact that the anomalous terms are higher-dimensional and non–renormalizable. The figure clearly shows that it is very difficult to observe the deviations due to the anomalous parameters $\kappa_\gamma$ and $\lambda_\gamma$ near $x_\gamma = 1 - m_Z^2/s$ where the Z–exchange contributions dominate over the W– exchange ones. Therefore, it is crucial to apply appropriate photon energy cuts to enhance the possibility to see the anomalous effects.

### III. NEUTRALINO–ASSOCIATED SINGLE–PHOTON PROCESS

Supersymmetry is a new symmetry which provides a well–motivated extension of the SM with an elegant solution to the gauge hierarchy problem. Most supersymmetry theories assume the so–called R-parity under which the SM particles are even and the supersymmetric particles are odd. The conservation of R–parity ensures the stability of the LSP so that it escapes from the detection. In most supersymmetric models, the lightest neutralino $\tilde{\chi}_1^0$ is the LSP in a wide range of parameter space. Because of the elusive property, the existence of the lightest neutralino can not be checked through the simplest process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ leaving no signals in a detector. However, the production of a lightest neutralino pair accompanied by a single photon in $e^+e^-$ collisions can give useful information on the existence of the LSP through the photon energy and angular distributions along with tuning the electron beam polarization and/or measuring the outgoing photon polarization.

In this section, we concentrate on the single–photon process $e^+e^- \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ in the MSSM. Because of the electroweak gauge symmetry breaking, the gauginos, the superpartners of gauge bosons, and the higgsinos, the superpartners of the Higgs bosons, can mix to give physical mass eigenstates in the MSSM. In particular, the photino $\tilde{\gamma}$ and the Zino $\tilde{Z}$ mix with two neutral higgsinos $\tilde{H}_1^0$ and $\tilde{H}_2^0$ to form four neutralino mass eigenstates $\tilde{\chi}_i^0$ [$i = 1$ to 4]. The neutralino masses and the mixing angles are determined by $m_Z$, $\tan\beta$, two soft SUSY–breaking gaugino mass parameters $M_1$ and $M_2$ and the SUSY–preserving higgsino mass parameter $\mu$. The symmetric $4 \times 4$ neutralino mass matrix can be diagonalized by a $4 \times 4$ unitary matrix $N$ [19]. Despite the involved neutralino mixing as well as the large number of Feynman diagrams, we will show that the production amplitude for the process $e^+e^- \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ can be also organized into the unified form in eq. (3), which enables us to investigate the dependence of the energy and angular spectrum of the outgoing photon on the relevant SUSY parameters.

The reaction $e^+e^- \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ in the MSSM involves 14 Feynman diagrams as depicted in Fig. 4. The selectron-exchange diagrams with the primed indices [Figs. (c')-(h')] are allowed

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[2]The conservative ranges of the parameters $\kappa_\gamma$ and $\lambda_\gamma$ quoted in Ref. [16] are considered.

due to the Majorana property of neutralinos, of which the amplitudes are related to those of the corresponding $t$-channel ones by

$$\mathcal{M}'_x(k_1, k_2) = -\mathcal{M}_x(k_2, k_1) \quad [x = c, d, e, f, g, h] \,, \tag{15}$$

where $k_1$ and $k_2$ are the four-momenta of the two lightest neutralinos. Due to the Majorana condition in eq. (8) the diagrams (A) and (B) can be expressed by

$$\mathcal{M}_{A,B} = \mathcal{M}_{a,b}(k_1, k_2) - \mathcal{M}_{a,b}(k_2, k_1) \equiv \mathcal{M}_{a,b}(k_1, k_2) + \mathcal{M}'_{a,b}(k_1, k_2) \,. \tag{16}$$

Defining the following combination to be $\mathcal{M}_L$:

$$\mathcal{M}_L \equiv \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}'_c + \mathcal{M}'_d + \mathcal{M}'_e + \mathcal{M}_f + \mathcal{M}_g + \mathcal{M}_h \,, \tag{17}$$

we can show that the sum of the remaining amplitudes, denoted by $\mathcal{M}_R$, satisfies the relation

$$\mathcal{M}_R(k_1, k_2) = -\mathcal{M}_L(k_2, k_1) \,, \tag{18}$$

and thus the total production amplitude $\mathcal{M}$ for the reaction $e^+ e^- \to \gamma \tilde{\chi}_1^0 \tilde{\chi}_1^0$ is given by

$$\mathcal{M} = \mathcal{M}_L + \mathcal{M}_R = \mathcal{M}_L(k_1, k_2) - \mathcal{M}_L(k_2, k_1) \,. \tag{19}$$

Then, the Fierz rearrangement formulas in eq. (2) cast the production amplitude into the unified form in eq. (3) with the following modifications:

$$
\begin{aligned}
g_{Z\tilde{\chi}_1^0\tilde{\chi}_1^0} &= \frac{1}{2} \left[ |N_{13}|^2 - |N_{14}|^2 \right] \,, \\
L_1 &= \epsilon_{_L} - \frac{1}{2} \left[ (p_1 - k_2)^2 - m_{\tilde{e}_L}^2 \right] f_{\tilde{e}_L} \,, \quad L_2 = \epsilon_{_L} - \frac{1}{2} \left[ (p_2 - k_1)^2 - m_{\tilde{e}_L}^2 \right] f_{\tilde{e}_L} \,, \\
R_1 &= \epsilon_{_R} + \frac{1}{2} \left[ (p_1 - k_1)^2 - m_{\tilde{e}_R}^2 \right] f_{\tilde{e}_R} \,, \quad R_2 = \epsilon_{_R} + \frac{1}{2} \left[ (p_2 - k_2)^2 - m_{\tilde{e}_R}^2 \right] f_{\tilde{e}_R} \,, \\
A_L^{\mu\nu} &= g^{\mu\nu} (k_2 - p_1) \cdot \varepsilon^* f_{\tilde{e}_L} \,, \quad A_R^{\mu\nu} = g^{\mu\nu} (k_2 - p_2) \cdot \varepsilon^* f_{\tilde{e}_R}
\end{aligned} \tag{20}
$$

where the form factors $f_{\tilde{e}_L}$ and $f_{\tilde{e}_R}$ describing the selectron-exchanges are given by

$$
\begin{aligned}
f_{\tilde{e}_L} &= \frac{4 \cos^2 \theta_W |g_L|^2}{g_{Z\tilde{\chi}_1^0\tilde{\chi}_1^0}} \frac{(k_1 + k_2)^2 - m_Z^2}{[(p_1 - k_2)^2 - m_{\tilde{e}_L}^2][(p_2 - k_1)^2 - m_{\tilde{e}_L}^2]} \,, \\
f_{\tilde{e}_R} &= \frac{4 \cos^2 \theta_W |g_R|^2}{g_{Z\tilde{\chi}_1^0\tilde{\chi}_1^0}} \frac{(k_1 + k_2)^2 - m_Z^2}{[(p_1 - k_1)^2 - m_{\tilde{e}_R}^2][(p_2 - k_2)^2 - m_{\tilde{e}_R}^2]} \,,
\end{aligned} \tag{21}
$$

with $g_L = (N_{12} + \tan \theta_W N_{11})/2$ and $g_R = \tan \theta_W N_{11}$. The factorization of the neutral vector currents of invisible neutralinos occurs again at the amplitude level. Compared to the amplitudes of the neutrino-associated processes in (4) and (14), we observe that the V+A structure of the electron current undergoes considerable changes due to the existence of the right–handed selectron exchanges. As a result, the use of the right–handed electron beam may be very helpful to reduce the SM background effects. This feature will be quantitatively demonstrated in the next section.

In Fig. 5, we have demonstrated the differential cross section with respect to the photon energy fraction $x_\gamma$ at $\sqrt{s} = 200$ GeV and 500 GeV for $\tan \beta = 2$ and 30, respectively, taking

$m_{\tilde{e}_{L,R}} = 100$ GeV. The lightest neutralino mass and the elements of the mixing matrix $N$ are computed by using $M_1 = 100$ GeV, $\mu = 100$ GeV, and the assumption of the gaugino mass unification condition $M_1 = (5/3)\tan^2\theta_W M_2$. We note that for $\tan\beta = 30$ the resonance peak around the Z–resonance pole is absent which is apparently present for $\tan\beta = 2$. These different behaviors according to $\tan\beta$ can be explained by comparing the maximally allowed photon energy $x_\gamma^{\max}$ with the photon energy fraction for the resonance peak $x_\gamma^{Z-peak}$. The maximum energy fraction of the photon corresponds to the largest momentum which is obtained when the photon is scattered against the collinear neutralinos:

$$x_\gamma^{\max} = 1 - \frac{4m_{\tilde{\chi}_1^0}^2}{s}. \tag{22}$$

Since the resonance peak occurs at $x_\gamma^{Z-peak} = 1 - m_Z^2/s$, there is no peak if $m_{\tilde{\chi}_1^0} \geq m_Z/2$. With the above numerical values for $M_1$, $M_2$, $\mu$, we have $m_{\tilde{\chi}_1^0} = 39$ GeV for $\tan\beta = 2$ and $m_{\tilde{\chi}_1^0} = 61$ GeV for $\tan\beta = 30$, which correctly explains the different behaviors. Therefore, a precise confirmation of the existence of the resonance peak after subtracting the SM background effects can provide valuable information on the lightest neutralino mass $m_{\tilde{\chi}_1^0}$ in the process $e^+e^- \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$.

## IV. LEFT-RIGHT ASYMMETRIES AND PHOTON POLARIZATION

One crucial difference of the neutralino–associated process from the neutralino–associated one is the existence of the right–handed selectron–exchanges, so that the ratio of the production rate with the right–handed electron beam to that with the left–handed one can be substantially large. Since a highly polarized electron beam with its beam polarization more than 90% is expected at future $e^+e^-$ linear colliders [20], it will be valuable to study the left–right asymmetries in identifying the origin of the single–photon events. Moreover, it is expected that the circular polarization of the outgoing photon is different. In this light, we present a quantitative analysis for the left–right asymmetries and the photon circular polarization in the single–photon processes $e^+e^- \to \gamma\nu\bar{\nu}$ and $e^+e^- \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$.

In order to measure a left–right asymmetry $A_{LR}$ defined by

$$A_{LR} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}, \tag{23}$$

we have only to switch the longitudinal electron polarization, which should be straightforward in a $e^+e^-$ linear collider. In order to measure the circular polarization of the final photon beam, we use a general method [21] which can be applied to any process producing a single photon. In the general formalism, the circular polarization is described by a Stokes' parameter $\xi_2$ that is nothing but the rate asymmetry:

$$\xi_2 = \frac{N_+ - N_-}{N_+ + N_-}, \tag{24}$$

where $N_\pm$ is the number of produced photons with positive and negative helicities.

Figure 6 shows the left–right asymmetries $A_{LR}$ as a function of the photon scattering angle $\theta$ at $\sqrt{s} = 200$ and 500 GeV with the same SUSY parameters as in Fig. 5. The

upper frame in the figure is for the neutrino–associated process, while the middle and lower frames are for the neutralino–associated ones for $\tan\beta = 2$ [middle] and $\tan\beta = 30$ [lower]. Clearly, the left–right asymmetries are very different in two processes; the asymmetries for the neutralino–associated process are always larger and even positive for $\sqrt{s} = 500$ GeV. Moreover, the dependence of the asymmetry on $\tan\beta$ becomes significant at $\sqrt{s} = 200$ GeV. As discussed in the previous section, the right–handed electron beam is very useful to identifying the neutralino–associated process, removing the large portion of the SM background.

In Fig. 7 we show the circular polarization degree $\xi_2$ of the outgoing photon as a function of the photon scattering angle $\theta$ for the neutrino–associated process [(a) and (c)] and the neutralino–associated one [(b) and (d)] with the same SUSY parameters as in Figs. 5 and 6. We set the electron beam to be purely left–handed in (a) and (b) and right–handed in (c) and (d). For the left–handed [right–handed] electron beam, $\xi_2$ is negative [positive] in the forward direction and positive [negative] in the backward direction, respectively. Note that the circular polarization in the neutralino–associated process is more sensitive to the beam energy of the right–handed electron beam than of the left–handed electron beam. This dependence is, however, opposite in the neutrino–associated process.

## V. CONCLUSIONS

We have studied in detail the single–photon events in high–energy $e^+e^-$ collisions as attributing the missing energy to neutrinos in the SM including the effects of the anomalous WW$\gamma$ couplings, or to neutralinos in the MSSM, which are assumed to be the LSP. We have found that the transition amplitudes for both processes can be organized into a generic simplified, factorized form; each neutral V$\pm$A vector current of missing energy carriers is factorized out and all the characteristics for the reaction is solely included in the electron vector current.

The amplitude reduction procedure described in Section II.A allows us to give a unified description of a single–photon production with a Dirac–type or Majorana–type neutrino–pair and to easily confirm their identical characteristics in the observation supported by the so-called Practical Dirac–Majorana Confusion Theorem. The generic amplitude form is preserved with the anomalous WW$\gamma$ couplings in the neutrino–associated process and it enables us to easily understand large contributions of the anomalous P– and C–invariant WW$\gamma$ couplings at higher energies and, in particular, at the points away from the Z peak. The neutralino–associated single–photon process in the MSSM involves the modification in both the left–handed and right-handed electron currents due to the left–handed and right-handed selectron exchanges. Nevertheless, the basic simplified amplitude form can be applied to the production process of two identical neutralinos of Majorana type as well. We have found that, due to these distinct properties, utilizing the left–right asymmetries for the longitudinal electron polarization and/or measuring the circular polarization of the outgoing photon may be very useful in disentangling the neutralino–associated processes from the neutrino–associated ones.

## ACKNOWLEDGMENTS

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# FIGURE CAPTIONS

**Fig. 1** Feynman Diagrams contributing to the neutrino–associated single–photon process $e^+e^- \to \nu\bar{\nu}\gamma$ in the SM.

**Fig. 2** The differential cross section of the neutrino–associated process $e^+e^-\gamma \to \nu\bar{\nu}$ with respect to the energy fraction of the outgoing photon $x_\gamma$. The solid line is for $\sqrt{s} = 200$ GeV and the dotted line for $\sqrt{s} = 500$ GeV.

**Fig. 3** The photon energy distributions $\mathrm{d}\sigma/\mathrm{d}x_\gamma$ of the process $e^+e^- \to \gamma\nu\bar{\nu}$ for the $e^+e^-$ c.m. energy of 200 GeV and 500 GeV with the different values for the anomalous parameters, $\kappa_\gamma$ and $\lambda_\gamma$. In (a) and (c) the value of $\lambda_\gamma$ is taken to be 0 and in (b) and (d) the value of $\kappa_\gamma$ taken to be 1.

**Fig. 4** Feynman diagrams contributing to the neutralino–associated single–photon process $e^+e^- \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ in the MSSM.

**Fig. 5** The different cross section of the process $e^+e^- \to \gamma\tilde{\chi}_1^0\tilde{\chi}_1^0$ with respect to the photon energy fraction $x_\gamma$. The lightest neutralino mass and its couplings are calculated with $M_1 = 100$ GeV, $\mu = 100$ GeV and the gaugino unification condition $M_1 = (5/3)\tan^2\theta_W M_2$ for two $\tan\beta$ values; $\tan\beta = 2$ in the left frame and $\tan\beta = 30$ in the right frame. The masses for the right–handed and left–handed selectrons are assumed to be 100 GeV.

**Fig. 6** The left–right asymmetries as a function of the photon scattering angle $\theta$ at $\sqrt{s} = 200$ and 500 GeV. The upper frame is for the neutrino–associated process, while the middle and lower frames are for the neutralino–associated processes for $\tan\beta = 2$ [middle] and $\tan\beta = 30$ [lower]. The other SUSY parameters are taken to be the same as in Fig. 5.

**Fig. 7** The degree of photon circular polarization as a function of the photon scattering angle $\theta$ for the neutrino–associated process [(a) and (c)] and the neutralino–associated process [(b) and (d)]. The electron beam is purely left–handed in (a) and (b), and right–handed in (c) and (d). The other SUSY parameters have the same values as in Fig. 5.
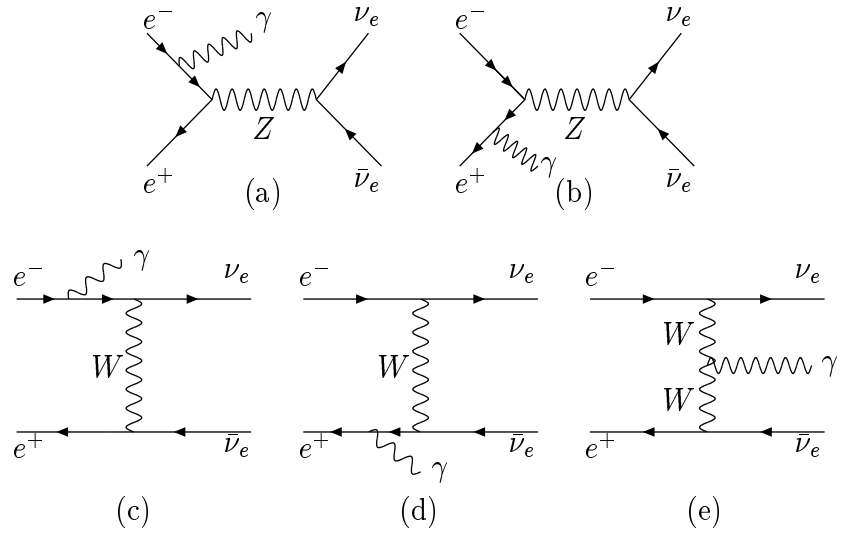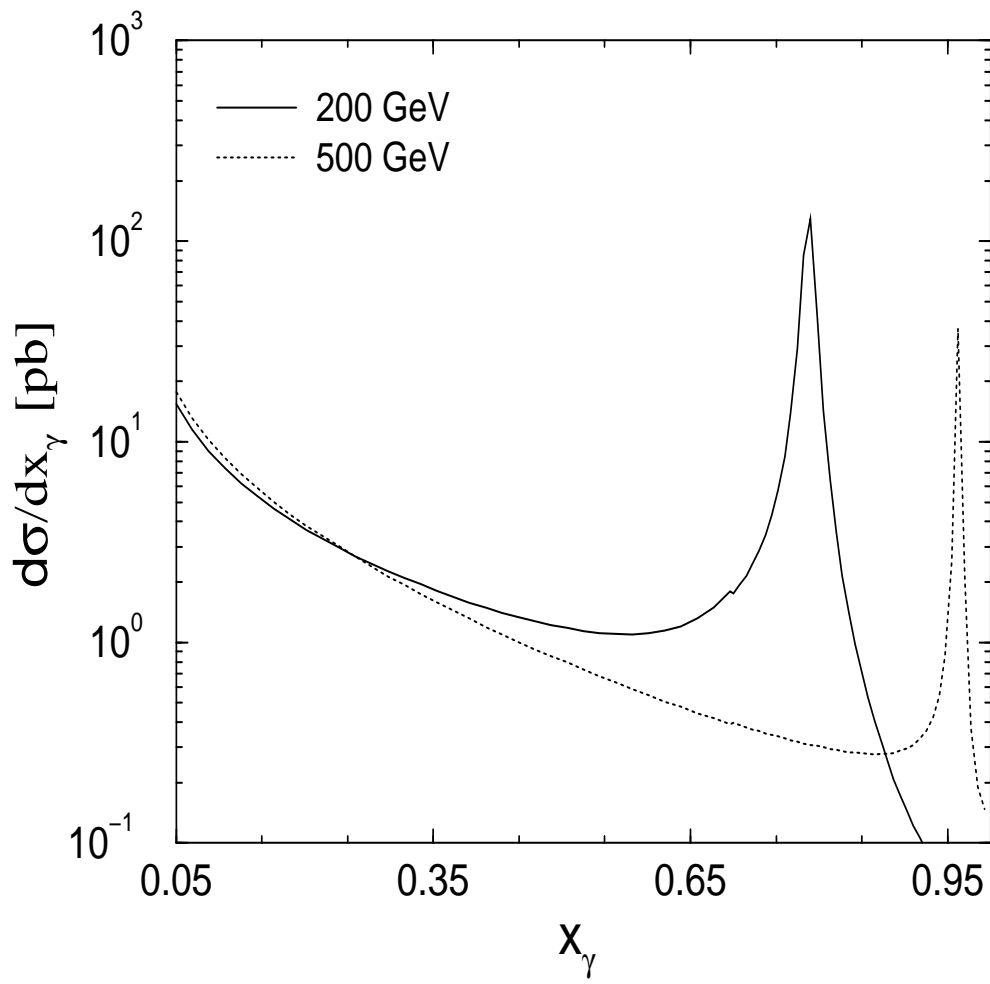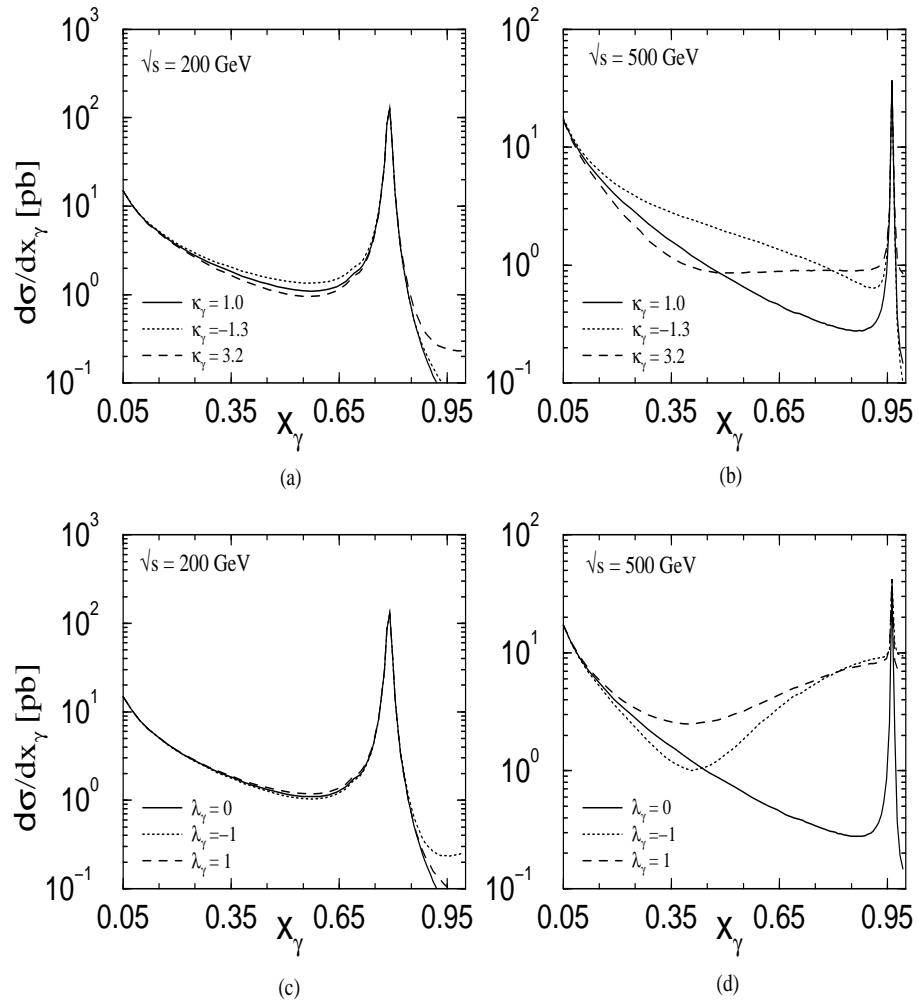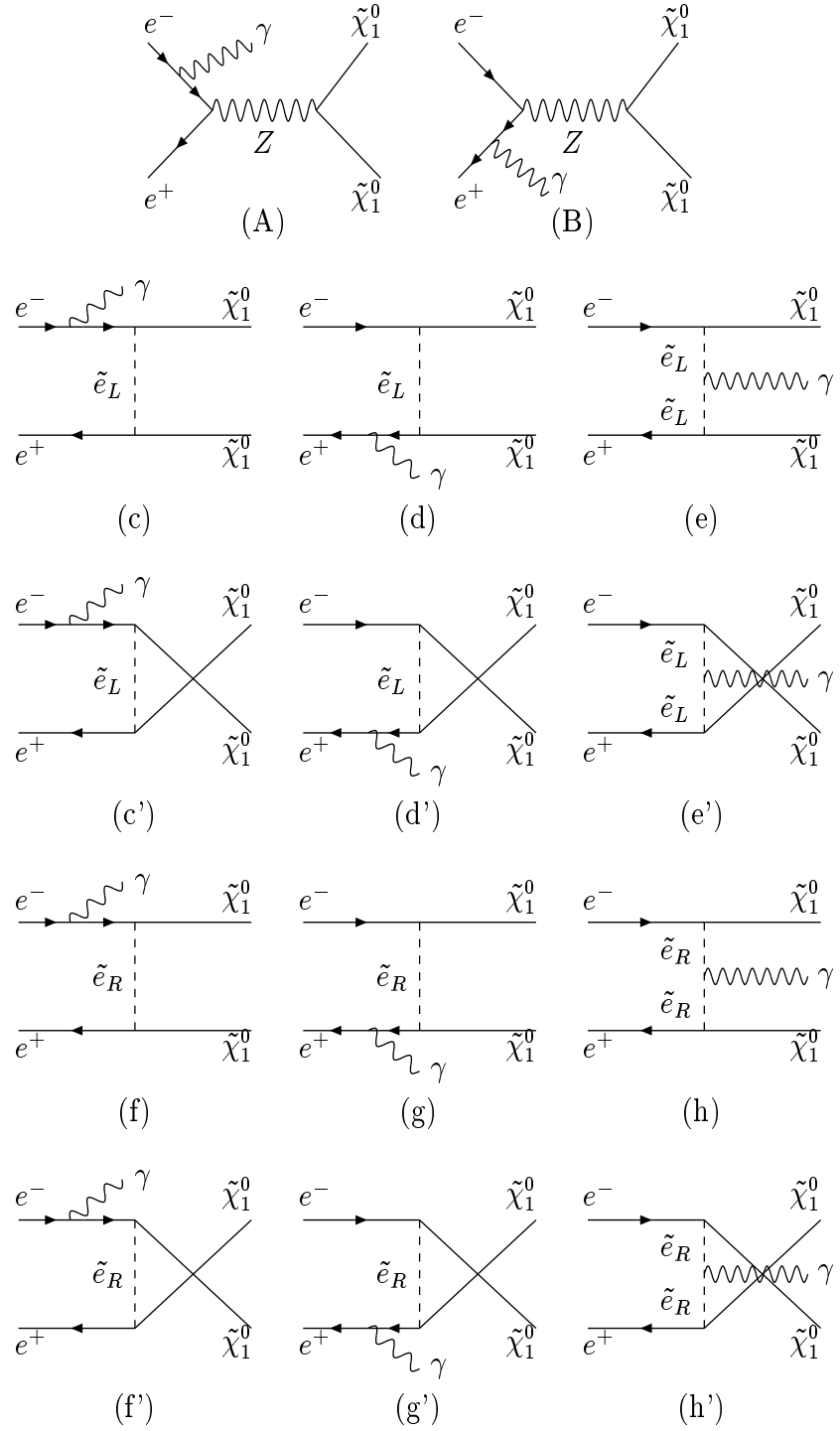
Figure 1

Figure 2

Figure 3

16

$e^-$ $\gamma$ $\tilde{\chi}^0_1$
$e^+$ $Z$ $\tilde{\chi}^0_1$
(A)

$e^-$ $\tilde{\chi}^0_1$
$e^+$ $\gamma$ $Z$ $\tilde{\chi}^0_1$
(B)

$e^-$ $\gamma$ $\tilde{\chi}^0_1$
$\tilde{e}_L$
$e^+$ $\tilde{\chi}^0_1$
(c)

$e^-$ $\tilde{\chi}^0_1$
$\tilde{e}_L$
$e^+$ $\gamma$ $\tilde{\chi}^0_1$
(d)

$e^-$ $\tilde{\chi}^0_1$
$\tilde{e}_L$ $\gamma$
$\tilde{e}_L$
$e^+$ $\tilde{\chi}^0_1$
(e)

$e^-$ $\gamma$ $\tilde{\chi}^0_1$
$\tilde{e}_L$
$e^+$ $\tilde{\chi}^0_1$
(c')

$e^-$ $\tilde{\chi}^0_1$
$\tilde{e}_L$
$e^+$ $\gamma$ $\tilde{\chi}^0_1$
(d')

$e^-$ $\tilde{\chi}^0_1$
$\tilde{e}_L$ $\gamma$
$\tilde{e}_L$
$e^+$ $\tilde{\chi}^0_1$
(e')

$e^-$ $\gamma$ $\tilde{\chi}^0_1$
$\tilde{e}_R$
$e^+$ $\tilde{\chi}^0_1$
(f)

$e^-$ $\tilde{\chi}^0_1$
$\tilde{e}_R$
$e^+$ $\gamma$ $\tilde{\chi}^0_1$
(g)

$e^-$ $\tilde{\chi}^0_1$
$\tilde{e}_R$ $\gamma$
$\tilde{e}_R$
$e^+$ $\tilde{\chi}^0_1$
(h)

$e^-$ $\gamma$ $\tilde{\chi}^0_1$
$\tilde{e}_R$
$e^+$ $\tilde{\chi}^0_1$
(f')

$e^-$ $\tilde{\chi}^0_1$
$\tilde{e}_R$
$e^+$ $\gamma$ $\tilde{\chi}^0_1$
(g')

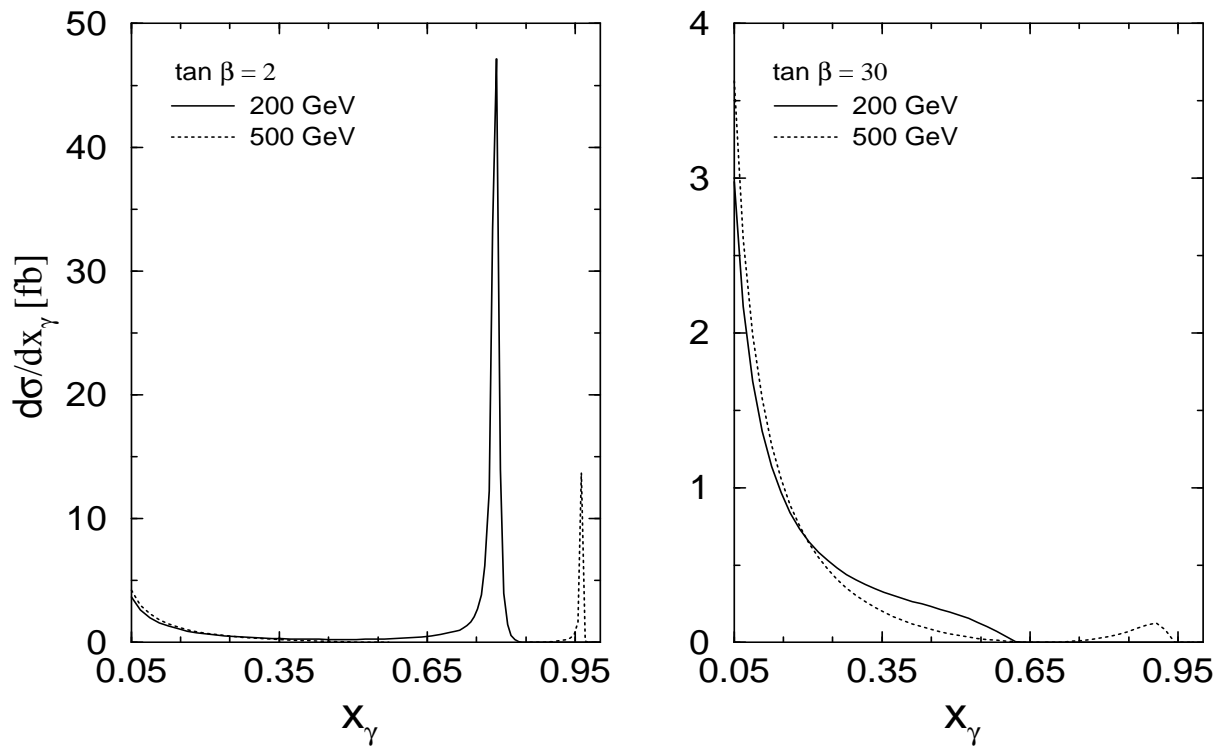$e^-$ $\tilde{\chi}^0_1$
$\tilde{e}_R$ $\gamma$
$\tilde{e}_R$
$e^+$ $\tilde{\chi}^0_1$
(h')

Figure 4

17

Figure 5

Figure 6

Figure 7